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12, 36; 58, 47
14, 38; 25, 67
15, 26; 34, 78

13, 28; 45, 67
15, 27; 38, 46
17, 25; 36, 48

12, 35; 46, 78
14, 26; 37, 58
18, 27; 34, 56

13, 26; 48, 57
14, 27; 35, 68
16, 23; 45, 78

13, 25; 47, 68
15, 23; 48, 67
16, 24; 38, 57

15, 24; 68, 37
16, 28; 35, 47
18, 23; 57, 46

12, 34; 57, 68
13, 24; 56, 78
17, 26; 38, 45

12, 37; 48, 56
14, 28; 36, 57
18, 24; 35, 67

with cyclic permutations of the last three columns;

12, 67; 35, 48
13, 46; 25, 78
15, 47; 23, 68

12, 45; 36, 78
14, 56; 28, 37
17, 68; 25, 34

12, 38; 47, 56
17, 23; 46, 58
18, 45; 27, 36

13, 27; 45, 68
14, 23; 58, 67
17, 56; 28, 34

13, 58; 26, 47
15, 28; 34, 67
18, 37; 25, 46

14, 78; 26, 35
16, 25; 38, 47
17, 58; 24, 36

15, 36; 27, 48
16, 37; 24, 58
17, 35; 28, 46

16, 27; 34, 58
17, 28; 36, 45
18, 47: 23, 56

16, 34; 27, 58
17, 24; 38, 56
18, 26; 37, 45

16, 58; 23, 47
17, 34; 28, 56
18, 25; 37, 46

16, 48; 25, 37
17, 46; 23, 58
18, 36; 27, 45

This solution, which I made in 1905, has been re-checked at two different times. Note that the first 24 orders furnish a Thompson program for 24 evenings, each couple to play against the other three couples in turn. It would be interesting to know whether or not 24 is a maximum* in the Thompson problem.

134. **Proposed by O. E. GLENN, Ph. D., Springfield, Mo.**

How many sets of solutions has the congruence $\lambda+\mu+\nu+\xi\equiv0 \pmod{p-1}$ p being a prime number; the order of λ, μ, ν, ξ being disregarded.

Solution by the PROPOSER.

Assume that $p>5$, and let n_i be the number of solutions in which i of the λ, μ, ν, ξ are congruent to each other (mod $p-1$). If $p\equiv1$ (mod 4), then $n_4=4$, viz.,

$$\lambda\equiv\mu\equiv\nu\equiv\xi\equiv0,\ \frac{p-1}{4},\ \frac{p-1}{2},\ \frac{3(p-1)}{4}.$$

If $p\equiv3$ (mod 4), $n_4=2$, viz., $\lambda\equiv\mu\equiv\nu\equiv\xi\equiv0,\ \dfrac{p-1}{2}$.

Next, let $i=3$, so that the congruence reduces to $\lambda+3\nu\equiv0 \pmod{p-1}$.

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*Note that a Thompson program for 22 evenings is given by (I)—(VIII) and

13, 25, 46, 78; 13, 26, 47, 58; 13, 27, 48, 56; 14, 25, 37, 68;
14, 26, 38, 57; 15, 24, 36, 78; 15, 27, 34, 68; 16, 28, 34, 57;
17, 24, 38, 56; 17, 26, 35, 48; 17, 28, 36, 45; 18, 23, 46, 57;
18, 24, 35, 67; 18, 25, 36, 47; 18, 26, 37, 45.

These 22 orders are mutually consistent, while no other is consistent with them.

Evidently ν cannot be congruent to $0, \frac{p-1}{4}, \frac{p-1}{2}$ or $\frac{3(p-1)}{4}$, for then $\nu \equiv \lambda$. Otherwise ν can have any value (mod $p-1$), so that when $p \equiv 1$ (mod 4), $n_3 = p - 1 - 4 = p - 5$, and when $p \equiv 3$ (mod 4), $n_3 = p - 3$.

When $i=2$, there are two possibilities.

(1) Let the parameters be congruent in pairs so that

$$2\lambda + 2\nu \equiv 0 \pmod{p-1}.$$

Here, if $\nu \equiv 0$, then must $\lambda \equiv \frac{p-1}{2}$ and inversely. Let all values (mod $p-1$) be assigned to ν in succession. With each value of ν belongs $dv(2; p-1) = 2$ values of λ determined by the congruence; as ν assumes its values λ repeats these values in another order twice. Hence, removing the two excluded solutions $(0, 0)$, $\left(\frac{p-1}{2}, \frac{p-1}{2}\right)$, we have, if $p \equiv 1$ (mod 4),

$$n_{2_1} = \frac{2(p-1)}{2} - 2 = p - 3,$$

and if $p \equiv 3$ (mod 4),

$$n_{2_1} = \frac{2(p-1)}{2} + 1 - 2 = p - 2.$$

(2) Next suppose that two parameters are congruent and the other two incongruent, so that

$$\lambda + \mu + 2\nu \equiv 0 \pmod{p-1}.$$

For the two values $\nu \equiv 0$, $\nu \equiv \frac{p-1}{2}$ the congruence reduces to $\lambda + \mu \equiv 0$ (mod $p-1$). Then μ is not $\equiv 0$ or $\frac{p-1}{2}$, since then $\lambda \equiv \mu$. Apart from these two exceptions μ can have any value (mod $p-1$), and for each μ a λ is determined by the congruence. Thus the cases $\nu \equiv 0, \frac{p-1}{2}$ yield $\frac{2(p-3)}{2}$ solutions. Aside from the two exceptions given ν can have any value (mod $p-1$). We may now assign to μ $p-3$ values exclusive of the value ν_1 assigned to ν, and the value $p-1-3\nu_1$. Then λ receives all values (mod $p-1$) exclusive of the two $p-1-3\nu_1$ and ν_1, respectively. Two values of λ are congruent to μ and must be excluded; *e. g.*, $\lambda \equiv \mu \equiv \frac{p-1-2\nu_1}{2}$ and $\lambda \equiv \mu \equiv p-1-\nu_1$. Aside from these, λ's values are the values of μ in another order. Hence, since $(\lambda\, \mu\, \nu)$ and $(\mu\, \lambda\, \nu)$ constitute the same solution, we obtain $(p-3)\left(\frac{p-3}{2} - 1\right) = (p-3)\left(\frac{p-5}{2}\right)$ new solutions. Thus $n_{2_2} = p - 3 + (p-3)\left(\frac{p-5}{2}\right) = \frac{1}{2}(p-3)^2$.

Now if we have regard to the order of λ, μ, ν, ξ, there are $(p-1)^3$ ways of satisfying the original congruence. For each of three parameters can be assigned in $p-1$ ways, and the congruence determines the remaining one. Hence we have

$$(p-1)^3=n_4+{}_3C_4n_3+6n_{2_1}+2{}_2C_4n_{2_2}+{}_4P_4n_1,$$

$$(p-1)^3=n_4+4n_3+6n_{2_1}+12n_{2_2}+24n_1.$$

Substituting the values found above for n_{2_2}, n_{2_1}, n_3, n_4, we get, when $p\equiv1$ (mod 4),

$$n_1=\tfrac{1}{24}(p^3-9p^2+29p-21)$$

and when $p\equiv3$ (mod 4),

$$n_1=\tfrac{1}{24}(p^3-9p^2+29p-33).$$

Hence, when $p>5$ is a prime of the form* $4l+1$ the number (N) of sets of solutions of $\lambda+\mu+\nu+\xi\equiv0$ (mod $p-1$) is,

$$N=\tfrac{1}{24}(p^3-9p^2+29p-21)+\tfrac{1}{2}(p-3)^2+p-3+p-5+4$$
$$=\tfrac{1}{24}(p^3+3p^2+5p-9).$$

But when p is of the form $4l+3$,

$$N=\tfrac{1}{24}(p^2-9p^2+29p-33)+\tfrac{1}{2}(p-3)^2+p-2+p-3+2$$
$$=\tfrac{1}{24}(p^3+3p^2+5p+3).$$

GEOMETRY.

284. Proposed by **JOHN JAMES QUINN, Ph. D.**, Warren, Pa.

a) Suppose that two radii R and r, whose center is the origin, revolve with uniform angular velocities 3θ and θ, respectively. What is the equation of the locus of P, the projection parallel to the X axis of the extremity of the radius r on the radius R produced if necessary.

b) Apply this curve to the trisection of an angle.

c) Suppose the ratio of their velocities is $n\theta:\theta$. Show how we can effect the multisection of an angle.

Solution by **A. H. HOLMES**, Brunswick, Maine.

a) Take O, the center of the circle, radius a, as the origin of coördinates. Then taking any angle θ, we shall have $r\sin3\theta=a\sin\theta$.

$\therefore r=\dfrac{a\sin\theta}{\sin3\theta}$ is the equation of the locus of the point P.

b) Construct the curve $r=\dfrac{a\sin\theta}{\sin3\theta}$. On the circumference of the circle

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*The values $p=3$, 5 constitute exceptions in the method employed. The final result does not hold for $p=5$. But, by inspection, when $p=5$, $N=10$; and when $p=3$, $N=3$.